Greek Letters

ΔT	$= T_\infty - T_o$
ΔT_i	$= T_\infty - T_i$
ϵ	= mass fraction of third component in the bulk
η	= similarity variable = $y\sqrt{c/R}$
θ	= angle in spherical coordinates
Θ	= dimensionless temperature
Θ_l	$= (T_l - T_i)/(T_\infty - T_i)$
κ	= thermal diffusivity = $k/\rho C_p$
λ	= latent heat of vaporization
μ	= dynamic viscosity
ν	= kinematic viscosity = μ/ρ
ρ	= density
ϕ_{C_p}	= ratio $(C_p/C_{p\infty})$
ϕ_k	= ratio (k/k_∞)
ϕ_μ	= ratio (μ/μ_∞)
ϕ_ν	= ratio $(2\nu_\infty/\nu_l)$
ϕ_ρ	= ratio (ρ/ρ_∞)
Ψ	= stream function

Subscripts

e	= at outside edge of the continuous phase boundary layer
g	= gas
i	= at the dispersed-continuous phase interface
l	= liquid
l, i	= in the liquid at the interface
$,o$	= bulk condition in droplet
s	= at the s-surface
v	= vapor (steam)
v, i	= in the vapor at the interface
vg, i	= in the continuous phase at the interface
$_1$	= air
$_2$	= vapor or steam
$_3$	= noncondensable but absorable component
$_\infty$	= in the bulk phase, far away from the droplet

LITERATURE CITED

Batchelor, G. K., "On Steady Laminar Flow with Closed Stream Lines at Large Reynolds Number," *J. Fluid Mechanics*, **1**, 177 (1956).

Brignell, A. S., "Solute Extraction from an Internally Circulating Spherical Liquid Drop," *Int. J. Heat and Mass Transfer*, **18**, 61 (1975)..

Chao, B. T., "Transient Heat and Mass Transfer to a Translating Droplet," *J. Heat Transfer*, **91**, 273 (1969).

Chung, J. N., and P. S. Ayyaswamy, "The Effect of Internal Circulation on the Heat Transfer of a Nuclear Reactor Containment Spray Droplet," *J. Am. Nuc. Soc.*, **35**, No. 3, 603 (1977).

Chung, J. N., and P. S. Ayyaswamy, "Laminar Film Condensation Heat and Mass Transfer in the Vicinity of the Forward Stagnation Point of a Spherical Droplet Translating in a Ternary Mixture: Numerical and Asymptotic Solutions," *Int. J. Heat and Mass Transfer*, **21**, No. 10, 1309 (1978a).

Chung, J. N., and P. S. Ayyaswamy, "Material Transport with Steam Condensation on a Moving Spray Droplet Including the Effect of Internal Chemical Reaction," *Topics in Two-Phase Heat Transfer and Flow*, ed., Bankoff, S. G., 153, ASME, New York (1978b).

Ford, J. D., and A. Lekic, "Rate of Growth of Drops during Condensation," *Int. J. Heat and Mass Transfer*, **16**, 61 (1973).

Harper, J. F., and D. W. Moore, "The Motion of a Spherical Liquid Drop at High Reynolds number," *J. Fluid Mechanics*, **32**, 367 (1968).

Harper, J. F., "The Motion of Bubbles and Drops through Liquids," *Advances in Applied Mechanics*, **12**, 59 (1972).

Levich, V. G., V. S. Krylov, and V. P. Vorotolin, "On the theory of unsteady diffusion from moving drops," *Dokl. Akad. Nauk SSSR*, **161**, 648 (1965).

Prakash, S., and W. A. Sirignano, "Liquid Fuel Droplet Heating with Internal Circulation," *Int. J. Heat and Mass Transfer*, **21**, 885 (1978).

Ranz, W., and W. Marshall, "Evaporation from Drops," *Chem. Eng. Prog.*, **48**, 141, 173 (1952).

Ruckenstein, E., "Mass Transfer between a Single Drop and a Continuous Phase," *Int. J. Heat and Mass Transfer*, **10**, 1785 (1967).

Tanaka, M., "Heat Transfer of a Spray Droplet in a Nuclear Reactor Containment," *Nucl. Tech., J. Am. Nuc. Soc.*, **47**, No. 2, 268 (1980).

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Local Hold-Up and Liquid Velocity in Air-Lift Reactors

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Measurements of local hold-up and liquid recirculation rate in an air-lift reactor were performed with two types of gas spargers using a manometric technique. A simple exponential function correlated properly the liquid velocity measured to the gas flow rate. The local hold-up varied appreciably along the column and showed a maximum in most of the cases. A simple linear relationship correlated the local gas velocity with the total flow rate of the mixture.

SCOPE

Air-lift reactors have recently been the object of much attention, especially in relation to the production of single-cell proteins from hydrocarbons. The special feature that distinguishes an air-lift reactor from the more common one without mechanical agitation (bubble column) is the recirculation of the liquid through a downcomer that connects the gas-liquid separator on top of the main bubbling section or riser to its lower part. A net liquid flow develops in the riser and the downcomer. The main advantages of this design are the capacity for satisfying the very high oxygen demand of hydrocarbon fermentation (Wang, 1968), low energy input, especially in large configurations (Hatch, 1973; Legrys, 1977), and the possibility of easier removal of the heat generated in the fermentation process due to the configuration, especially in the case of external downcomers that act as heat exchangers (Schugerl et al., 1977).

The main difficulty in the mathematical representation of the air-lift fermentor is the definition of the hydrodynamics of the system. The liquid and gas velocities should appear in any material balance (Merchuk et al., 1980). Although experimental data and correlations are available in the literature on bubble columns, the air-lift fermentor differs because the liquid flow is not independent of the gas flow rate, but is in fact originated by it. The present work was undertaken with the aim of clarifying the interdependence of these two flows, and their influence on the gas hold-up along the bubbling section.

CONCLUSIONS AND SIGNIFICANCE

The measured values of the local hold-up depend on the type of sparger used for the gas and on the resistance to the fluid flow in the circuit. When this resistance is low and the gas is distributed evenly in the cross-section of the riser, the local hold-up rises linearly in the tube. This rise is significant, reaching 30% at high superficial gas velocities. When the liquid flow rate is hindered by a greater resistance, provoked in the reported experiments by partially closing the downcomer, or when a single-orifice gas sparger is used, the form of the hold-up profiles changes.

A maximum, which descends as the gas superficial velocity increases, appears at a given height above the sparger. The difference between local hold-ups may reach 80% in these cases. Since the appearance of such maxima is attributed to a larger concentration of big bubbles in the center of the tube (causing higher velocities in the axis as well as internal recirculation in the riser), the conditions that will give a regime similar to a plug-flow are: even initial gas distribution in the section of the riser, and high liquid velocities, which are obtained by minimizing the resistances to fluid flow in the circuit.

The liquid velocity was found to be a simple power law function of the gas flow rate (Eq. 24). The constants in this equation depend both on the geometry of the system and on the regime of the two-phase flow in the riser. The gas local velocity can be expressed by a single linear relationship as a function of the total superficial velocity, J_M, for all the geometries tested (Figure 13). This relation can be used, together with kinetic information, to compose a mathematical representation of a fermentation process in an air-lift reactor. Such a model is necessary for design, optimization and control of the process.

The application of bubble columns as industrial scale reactors has lately received much attention: a series of patents have been granted and many research works have been carried out (Schugerl et al., 1977). A case of special interest is the air-lift reactor, first described by Le Francois et al. (1955), which allows easy removal of the heat generated in the fermentation process in a simple way, provides oxygen at a satisfactory rate, and is convenient from the energy point of view due to its flow characteristics. This type of equipment is suited to the needs of protein production from hydrocarbon sources, and full-scale plants already exist in several countries (Schugerl et al., 1977). Fundamental studies were done on the subject by Hatch (1973), Hatch and Wang (1975), and Orazem and Erickson (1979), among others.

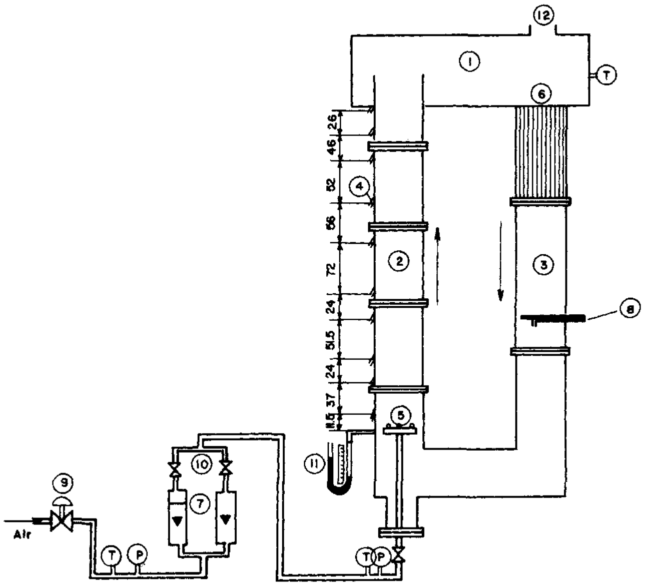

Figure 1. Experimental setup: 1. gas-liquid separator; 2. riser; 3. downcomer; 4. manometers; 5. sparger; 6. straightening vanes; 7. rotameters; 8. liquid flowmeter; 9. pressure controller; 10. control valves; 11. mercury manometer; 12. gas outlet. Dimensions are given in cm.

The problem of modeling an air-lift reactor has been considered by Hatch (1973), Ho et al. (1977) and Merchuk et al. (1980). The main difficulty in such modeling is the lack of information on the hydrodynamics of the air-lift reactor. Data and correlations can be found in the literature on gas hold-up and its relationship to gas and liquid velocities, but in the case of the air-lift reactor, these velocities are not independent. For a given system, once the air flow rate is set, the liquid flow rate as well as the hold-up becomes fixed. The variations of gas hold-up along the column and liquid velocities are studied here as a function of the gas flow rate for difference degrees of resistance to flow in the circuit.

EXPERIMENTAL APPROACH

The column in the experimental setup (Figure 1) was made of Plexiglas and had a total volume of about 0.3 m^3. The distance between the pressure taps in the riser is indicated in centimeters. The dimensions of the various parts of the equipment can be seen in Table 1.

Air was supplied at 6 atm pressure, and the maximum flow rate was 13.8 m^3/s. Gas sparger A was a 0.09-m diameter copper tube bent to form a 0.10-m diameter ring. Fourteen 0.025-m diameter holes were drilled in the upper side of the ring. Gas sparger B was a single orifice sparger 0.09 m in diameter.

The resistance to the liquid recirculation was changed by closing several of the tubes acting as straightening vanes in the downcomer. An Anubar 733-31655 liquid flowmeter was placed in the downcomer. The pressure along the riser, used for the hold-up calculations, was measured with reference to the pressure at the height of the sparger with differential manometers filled with CTC. The absolute pressure at this point was measured with a mercury manometer open to the atmosphere. Temperatures were measured both in the liquid and in the gas

TABLE 1. MAIN DIMENSIONS OF THE AIR-LIFT SYSTEM

Riser Diameter	0.14 m
Riser Height	4.05 m
Downcomer Diameter	0.14 m
Downcomer Height	4.05 m
Gas-Liquid Separator	0.5 × 0.5 × 0.7 m
Separation between Tubes	0.35 m
Straightening Vanes Length	1.5 m

before injection into the system, and they varied between 18 and 23°C. Separation in the top section was complete so that gas could not recirculate through the downcomer. Details of the operational procedure are given by Stein (1979).

HOLD-UP CALCULATIONS

Assuming one-directional isothermal flow, steady state, constant cross-section, negligible mass transfer effects between gas and liquid, and constant properties in a cross-section, the momentum balance equations for the liquid phase are given by Wallis (1969) as:

$$-\rho_L A_L g dz - A_L dp + (\tau_{GL} C_{GL} - \tau_{WL} C_{WL}) dz - \overline{d\dot{m}_L V_L} \tag{1}$$

Similarly, for the gas phase considered as a separate continuous phase:

$$-\rho_G A_G g dz - A_G dp + (\tau_{LG} C_{LG} - \tau_{WG} C_{WG}) dz = \overline{d\dot{m}_G V_G} \tag{2}$$

In this case,

$$A_L + A_G = A \tag{3}$$

$$C_{LG} = C_{GL} \tag{4}$$

$$\tau_{LG} = -\tau_{GL} \tag{5}$$

$$C_{WL} = \pi D_C dz \tag{6}$$

Introducing the gas hold-up as:

$$A_G = \phi A \tag{7}$$

and considering that the gas-wall friction is negligible with respect to the liquid-wall friction, a single expression that relates ϕ and V_L to the energy losses in the tube is obtained. Relating the liquid superficial velocity, $J_L = Q_L/A$, to the liquid flux V_L by:

$$V_L = \frac{J_L}{1-\phi} \tag{8}$$

we obtain:

$$-\frac{dp}{dz} = \frac{4\tau_{WL}}{D_C} + \rho_L J_L^2 \frac{d}{dz}\left(\frac{1}{1-\phi}\right) + \rho_L g(1-\phi) \tag{9}$$

Eq. 9 expresses the three components responsible for the pressure drop in the tube: friction, acceleration and gravitation.

Direct measurements were done on h, the height in the differential manometers, which is related to the pressure by:

$$(\rho_L - \rho_m) g \frac{dh}{dz} = \frac{dP}{dz} + \rho_L g \tag{10}$$

In order to express the frictional term, the following expression (Wallis, 1969) was used:

$$\tau_{WL} = \frac{1}{2} C_{fM} \rho_M J_M^2 \tag{11}$$

where

$$J_M = J_L + J_G \tag{12}$$

$$J_G = Q_G/A \tag{13}$$

$$\rho_M = \rho_L(1-\phi) + \rho_G \phi \tag{14}$$

Eq. 11 can be approximated as:

$$\tau_{WL} = \frac{1}{2} C_{fM} \rho_L J_L J_M \tag{15}$$

The coefficient C_{fM} is given by Nassos and Bankoff (1967) as:

$$C_{fM} = 0.046\, N_{Re}^{-0.2} \tag{16}$$

where the Reynolds number is given by:

$$N_{Re} = \frac{\rho_L J_L D_C}{(1-\phi)\mu_L} \tag{17}$$

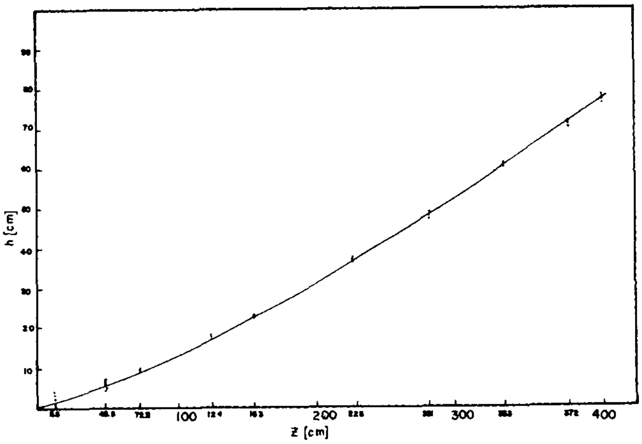

Figure 2. Experimental data of manometer heights along the column and the corresponding polynomial fit for a typical operational condition.

When Eqs. 10, 15, 16 and 17 are replaced in Eq. 9, an implicit expression of the gas hold-up can be obtained as:

$$\phi = \frac{0.092\, J_L^{0.8} J_M}{g(\nu_L)^{-0.2} D_C^{1.2}} (1-\phi)^{0.2} + \frac{J_L^2}{g(1-\phi)^2}\frac{d\phi}{dz} + \frac{(\rho_m - \rho_L)}{\rho_L}\frac{dh}{dz} \tag{18}$$

Here, dh/dz, the variation of the reading of the manometers with the height of the column, is obtained from the experimental data. It is the derivative of the polynomial fitting the data, as will be explained later on. The variation of hold-up along the height of the riser is evaluated from the second derivative of the experimental data taking:

$$\frac{d\phi}{dz} = \frac{(\rho_m - \rho_L)}{\rho_L}\frac{d^2h}{dz^2} \tag{19}$$

Eq. 19 is an approximation used to evaluate the hold-up derivative in Eq. 18, and implies that the contribution of the variations of the frictional and accelerational terms to the changes in hold-up along the column are negligible. The lower the gas flow rates, the better this approximation. Eq. 18 shows three terms representing, from left to right, friction, acceleration and static head. Their typical contributions will be 24%, 1% and 75% of ϕ, respectively. In the cases of very high gas flow rates, the acceleration term will rise to 2 or 3% of ϕ. Therefore, the approximation given by Eq. 19 was considered satisfactory.

The overall flux J_M is given by Eq. 12. In Eq. 12, J_L is constant along the column for given operational conditions, but J_G varies, increasing towards the top of the column. J_G can be calculated from the measurements in the rotameters and from the pressure and temperature for each height. Details are given by Stein (1979).

Each experimental point was repeated independently several times and a fourth-order polynomial was fitted to them. An example of the dispersion of the experimental points and the polynomial fitting for a typical operational condition can be seen in Figure 2. With the exception of the zone of small values of z near the gas sparger, the deviation of the experimental points is about 1%.

EXPERIMENTAL RESULTS

Local Hold-Ups

The local values of gas hold-up along the riser for both gas spargers used against gas flow rate are given in Figure 3 for the case of fully-open downcomer. A similar plot for the case where only 43% of the section of the downcomer is open to the recirculation of liquid is shown in Figure 4. This was obtained by closing several of the tubes that were used as straightening vanes in order to increase the resistance to the liquid recirculation and, in this way, change the liquid velocity.

Figures 3 and 4 show that the values of hold-up can increase up to 80% from the initial value at the bottom of the column in the range of

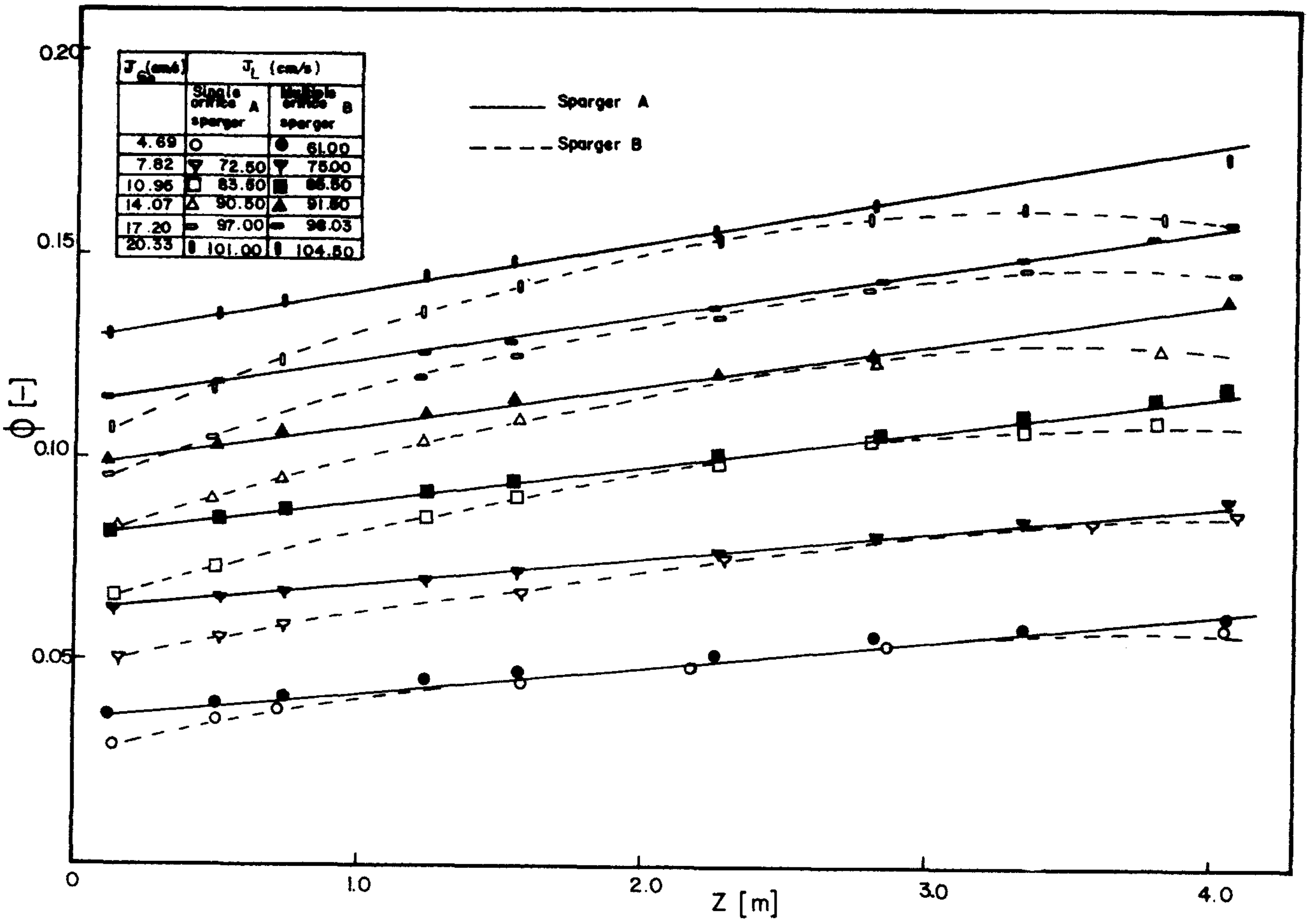

Figure 3. Dependence of the gas hold-up ϕ on distance from the sparger z for various superficial gas velocities and the downcomer fully open.

variables studied. Figure 3 corresponds to high values of liquid superficial velocities. A clear qualitative difference is seen between the data corresponding to sparger A, which gives an approximately even distribution of the gas in the section of the column, and the single-orifice sparger B. The data corresponding to sparger A increase almost linearly with z, while those corresponding to sparger B increase more sharply at low z, reach a maximum and then decrease. These maxima tend to appear earlier on z as the gas superficial velocity increases. The same phenomenon can be seen even more pronounced in Figure 4, where the partial closure of the downcomer causes lower liquid velocities. In this case, also sparger A gives profiles of ϕ that show a maximum.

The straight ϕ profiles for sparger A in Figure 3 indicate that coalescence is not important in this case. Bubbles ascend almost without interaction and the increase in hold-up indicates that the growing volume of the bubbles due to the decrease of pressure prevails over the increase in the associated ascending velocity. In addition, if a bubble expands too much, it will break into smaller bubbles that ascend more slowly without change in volume. On the other hand, if the rate of coalescence becomes important, the bubble size will grow without an associated increase in total gas volume and the hold-up will tend to decrease due to the higher bubble mean ascending velocity.

The single-orifice sparger creates an uneven distribution of gas in the section of the riser and the bubbles generated are larger. This provokes a higher mixture velocity in the center of the tube, and the large differences in liquid velocity induce a migration of the larger bubbles toward the axis. Thus, a high concentration of bubbles is created around the axis, which increases the coalescence. This phenomenon of enrichment of large bubbles in the center of the column is well-known (Calderbank et al., 1964; Ueyama and Miyauchi, 1977a). It has been used to explain the existence of a maximum in the overall column hold-up as a function of the superficial gas velocity which is obtained with a single-orifice sparger, and which does not appear with a perforated plate or porous sparger (Schugerl et al., 1977). Here, a maximum in the local hold-up at constant superficial velocity is reported.

An additional manifestation related to the same phenomenon is the generation of descending sidestreams near the walls, which appear in order to conciliate the high liquid velocity in the center with a given value of the net liquid flux (Ueyama and Miyauchi, 1977b). This is known to augment the overall hold-up and reaches its extreme at zero net liquid flow. Figure 4 shows that partial closure of the downcomer, which reduces the liquid flow and consequently increases the internal recirculation, leads not only to an increase of the hold-up, but also to an accentuation of the phenomenon of the maximum in the local values of hold-up. In this case, even the multiple-orifice sparger A gives maxima which, as before, tend to appear earlier on z as the gas superficial velocity increases. Sparger B, however, always gives this maximum at lower gas superficial velocities than does sparger A.

Mean Hold-Up

Figure 5 shows the correlation of the mean hold-up defined as:

$$\phi = \frac{1}{L}\int_0^L \phi(z)dz \tag{20}$$

versus the local hold-up at point $z = 2.25$ m, ϕ'. This was the pressure measuring tap located closer to the middle point of the column. It suggests that within the range of variables studied, ϕ' may be taken as representative of the mean hold-up in the riser.

Figure 6 shows the data of ϕ' as a function of the superficial gas velocity at $z = 2.25$ m, J_G' for several openings of the downcomer and sparger A. Similar data obtained with sparger B are shown in Figure 7. In both cases, the behavior is quite similar and concurs with the published data on bubble columns. It shows a sharp increase in the low range of superficial gas velocity, followed by a milder increase rate as J_G gets higher. No maxima such as those reported by Schugerl et al. (1977), among many others, were found within the range of flows explored. For

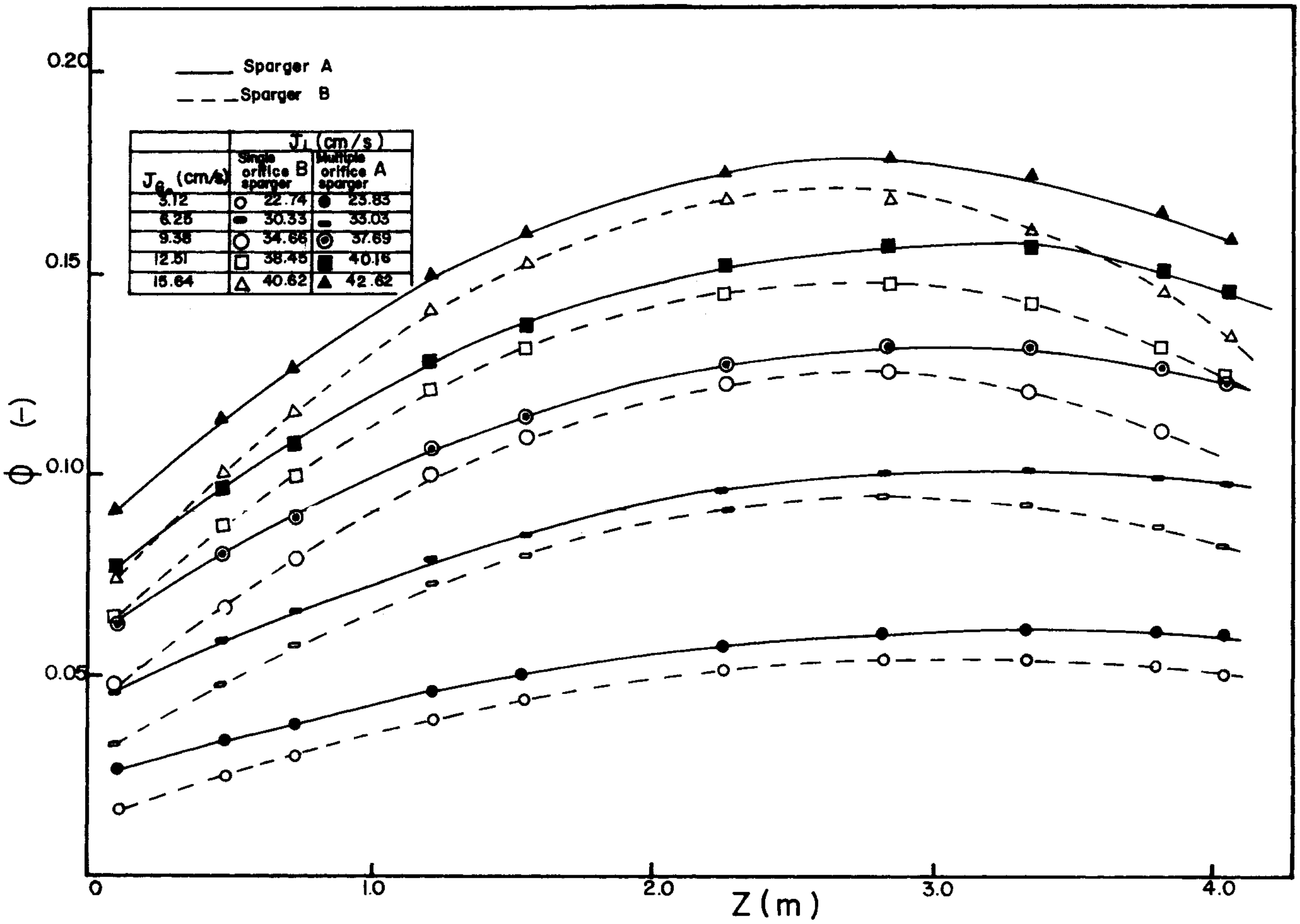

Figure. 4. Dependence of the gas hold-up ϕ on the distance from the sparger z for various superficial gas velocities and 43% of the downcomer section open.

both spargers, the influence of the closure of the downcomer, which decreases the liquid superficial velocity, is very strong, especially as the free section of the downcomer becomes smaller. The maximum values of ϕ' are found in both cases for a completely closed downcomer at zero liquid superficial velocity.

Figure 8 shows again the experimental data of Figures 6 and 7, but replotted with the liquid superficial velocity J_L as a parameter. This form of representation is very common in reporting data on bubble columns where J_L is independent of J_G. Figure 8 shows that the mean hold-up for all liquid superficial velocities is higher in the case of the multiple-orifice sparger A. This is in agreement with data reported by Freedman and Davidson (1969), and Schugerl et al. (1977), among others.

Absolute and Relative Gas Velocities

Figure 9 shows the absolute and relative gas velocities as a function of the gas superficial velocity for sparger A. Since all gas velocities vary along the riser, the point $z = 2.25$ m was selected as representative.

Zuber and Findley (1965) showed that in bubble flow the relative gas-liquid velocity is given by:

$$V_r = 1.53\ (\sigma g \Delta\rho/\rho_L^2)^{1/2} \tag{21}$$

This expression predicted for the air-water system a value of about 0.30 m/s, independent of the superficial gas velocity. On the other hand, for the bubble-slug region, they give

$$V_r = 0.35\ (g \Delta\rho D_c/\rho_L)^{1/2} \tag{22}$$

predicting for $D_c = 0.14$ m about 0.40 m/s.

Figure 9 shows that even if V_r remains in this range of values, an influence of J_G is present. The relative gas velocity obtained is in agreement with Zuber and Findley's predictions at low superficial gas velocities; but as J_G increases a minimum appears, and after it, higher values of V_r are obtained, increasing as the obstruction in the downcomer is increased (i.e., the liquid velocity is decreased). A similar observation was made by Schugerl et al. (1977) on their air-water system. They explained the phenomenon as one due to coalescence of

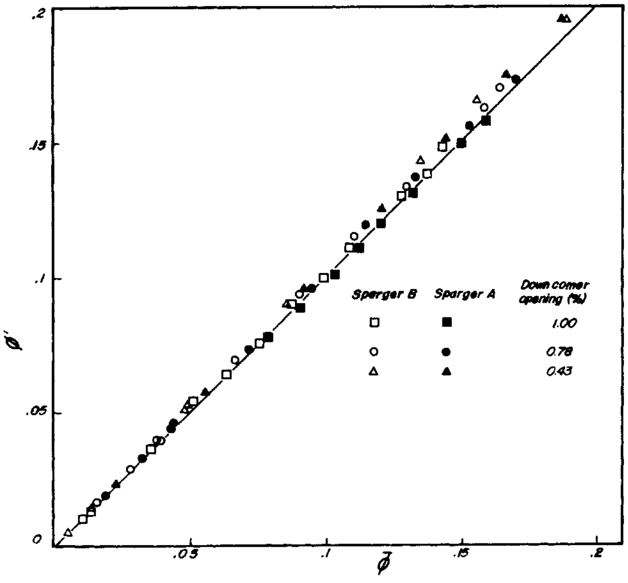

Figure 5. Mean hold-up ϕ vs. the local hold-up at z = 2.25 m, ϕ'.

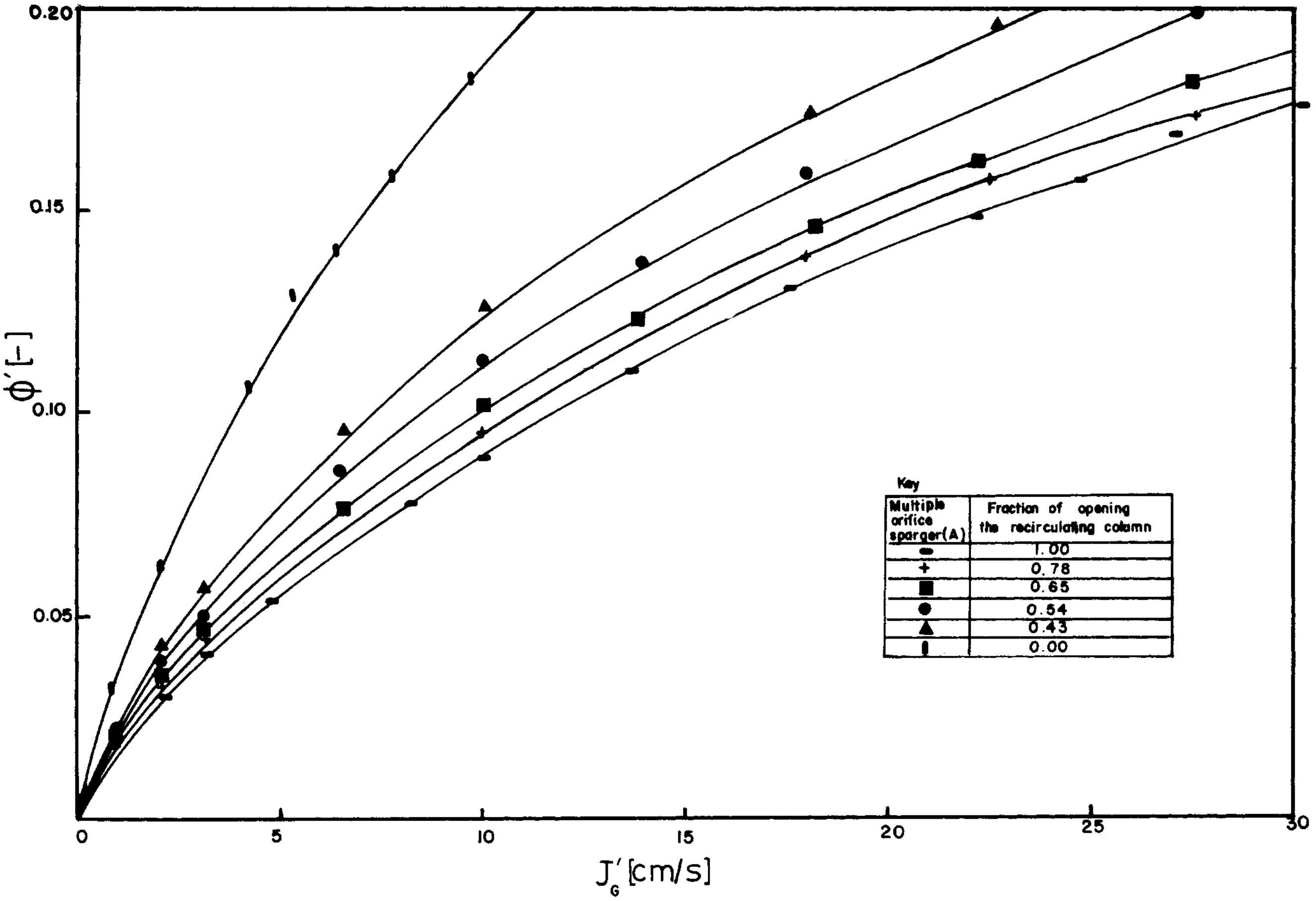

Figure 6. Variation of ϕ' with J'_G for various openings of the downcomer section, sparger A.

bubbles. On the other hand, they found that additives like alcohols, which hinder coalescence, produce curves of V_r vs. J_G that do not show a minimum, and concur with the expression given by Marucci (1969):

$$\frac{V_r}{J_G} = (1 - \phi)^2/(1 - \phi^{5/3}) \tag{23}$$

deduced for a coalescence-free system. The data reported in Figure 9 seem to indicate that the bubbles rise almost without coalescence at low superficial gas velocity; but at high values the coalescence phenomenon becomes more and more important and the behavior departs from that predicted by Marucci (1969). Coalescence reduces the air-lift effect, and the relative velocity increases.

The second influential factor is the presence of internal recirculation, which tends to increase the gas velocity at the axis, as noted before. This explains the higher values of V_r obtained as the downcomer is obstructed.

The curves of the absolute gas velocity show a quasi-exponential form in the range of net bubble flow (low J_G), and after about 0.05 m/s they increase almost linearly with the superficial gas velocity J_G, as can be seen in Figure 9.

Liquid Velocity

The liquid superficial velocity J_L is shown in Figure 10 as a function of the gas superficial velocity for several openings of the downcomer and for both spargers used. Since the superficial gas velocity J_G varies along the column, it was decided to adopt again a value of z = 2.25 m as representative.

The dependence of J_L on J_G is a distinctive characteristic of the air-lift reactor. We can see in Figure 10 that the liquid velocities decrease as the section of the downcomer is closed. In addition, the rate of growth of J_L with J_G is very high at low J_G and decreases afterwards, tending to an asymptotic value that is reached earlier as the downcomer is closed farther. This implies that the air-lift effect is less as we approach the slug flow regime, and that the asymptotic zone indicates a situation where, due to the frictional losses in the circuit, the energy brought by the gas stream is dissipated more and more in internal recirculation loops within the riser.

With Eq. 8, the liquid velocity V_L can be calculated from the measured values of J_L and the hold-up, ϕ.

Figure 11 shows the variation of V_L as a function of J_G for various openings of the downcomer and sparger A on a log-log scale. Straight lines were obtained, but broke around the transition point from bubble to turbulent bubble flow. For the case of total openings of the downcomer, the line does not break within the studied range of J_G. Figure 11 shows that the liquid velocity may be represented by equations of the type:

$$V_L = bJ_G^n \tag{24}$$

where the constants b and n will depend on the geometry of the air-lift reactor and need to be determined experimentally for each case. While b varies with the reactor configuration, n appears to be almost constant for a given flow regime. The change of n gives an objective method to recognize the onset of the turbulent bubble flow.

If the liquid velocity cannot be directly measured and if no correlation of the type of Eq. 24 is available, V_L can be obtained from an energy balance in the system if the hold-up is known. This approach will also make it possible to foresee the influence of the system geometry on V_L.

We have seen in Eq. 9 that the pressure drop in the reactor can be considered as generated by three components: gravitation, friction and acceleration. The last component will be neglected because calculations performed for our system have shown that the contribution of acceleration to the total pressure drop was in all cases less than 2%.

The pressure drop in the riser and in the downcomer are given respectively by Eqs. 25 and 26:

$$\Delta P_r = P_{1r} - P_{2r} = \Delta P_{Fr} + \Delta P_{Hr} \tag{25}$$

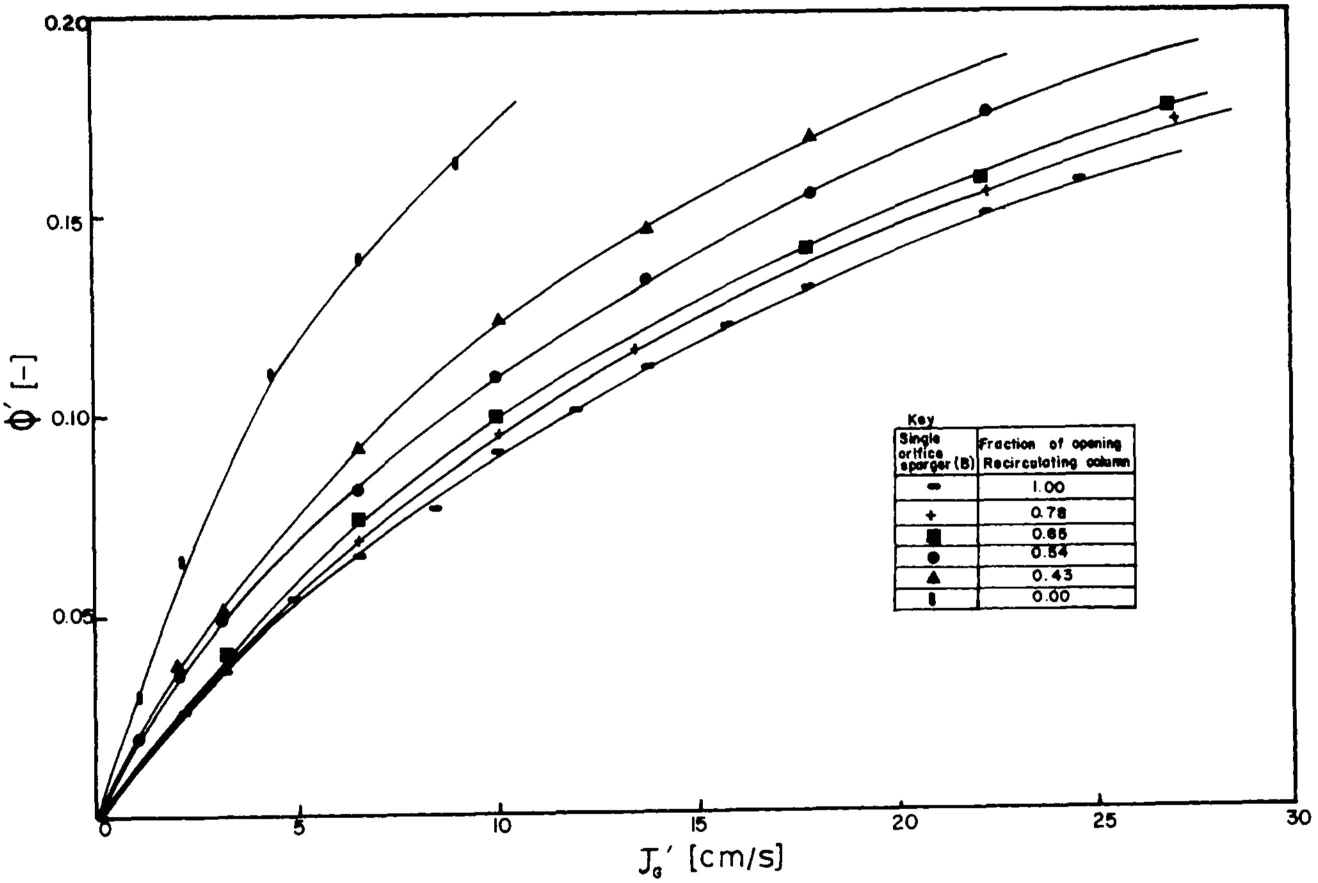

Figure 7. Variation of ϕ' with J_G' for various openings of the downcomer section, sparger B.

$$\Delta P_d = P_{1d} - P_{2d} = \Delta P_{Fd} + \Delta P_{Hd} \tag{26}$$

where the subindices indicate:

1 = bottom of the tubes
2 = top of the tubes
r = riser
d = downcomer
F = frictional loss
H = hydrostatic loss

Neglecting the pressure drop in the gas-liquid separator at the top of the equipment and indicating the pressure drop associated with the loop at the bottom and the straightening vanes as ΔP_B, Eqs. 25 and 26 give:

$$\Delta P_{Hd} - \Delta P_{Hr} = \Delta P_B + \Delta P_{Fr} + \Delta P_{Fd} \tag{27}$$

The left-hand term in Eq. 27 can be expressed as a function of the hold-up:

$$\Delta P_{Hd} - \Delta P_{Hr} = L\rho_L\bar{\Phi}g \tag{28}$$

The frictional terms for the riser can be represented by an integrated form of Eq. 15:

$$\Delta P_{Fr} = 2C_{fM}\rho_L J_L(J_L + J_G)\frac{L}{Dh} \tag{29}$$

with C_{fM} being given by Eq. 16.

For the downcomer and the additional term ΔP_B, it can be written respectively as:

$$\Delta P_{Fd} = 2C_{fL}\rho_L L J_L/D_{hd} \tag{30}$$

$$\Delta P_B = 2C_{fL}\rho_L L_E J_L/D_{hB} \tag{31}$$

L_E being an equivalent length of straight tube. The coefficient C_{fL} may be taken as the Blausius form:

$$C_{fL} = 0.0791\,Re^{-0.25} \tag{32}$$

Now Eq. 27 can be written for round tubes as:

$$\bar{\Phi} = \frac{2J_L^2}{gD_r}\left[\left(1 + \frac{J_{Gr}}{J_L}\right)C_{fM} + C_{fL}\left(1 + \frac{L_E}{L}\right)\zeta^{2.5}\right] \tag{33}$$

where ζ is the ratio of cross-sectional areas of the riser and the downcomer. Eq. 33 is quite general and permits calculation of the superficial liquid velocity from direct measurements of the superficial gas velocity and the hold-up for a given geometric system. As will be shown in Eq. 34, the hold-up can be evaluated by a suitable correlation as a function of the superficial gas and liquid velocities.

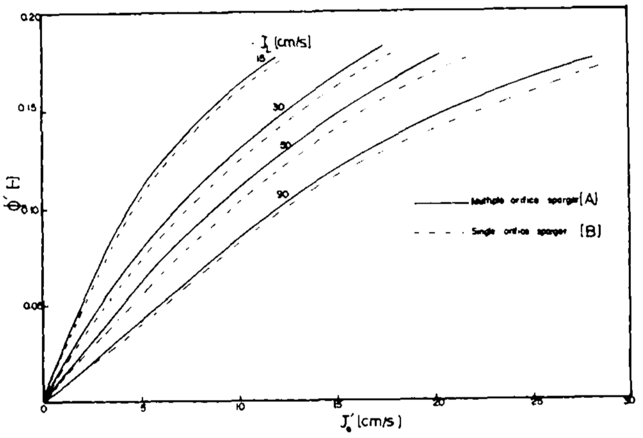

Figure 8. Variation of ϕ' with J_G' for various liquid flow rates, of spargers A and B.

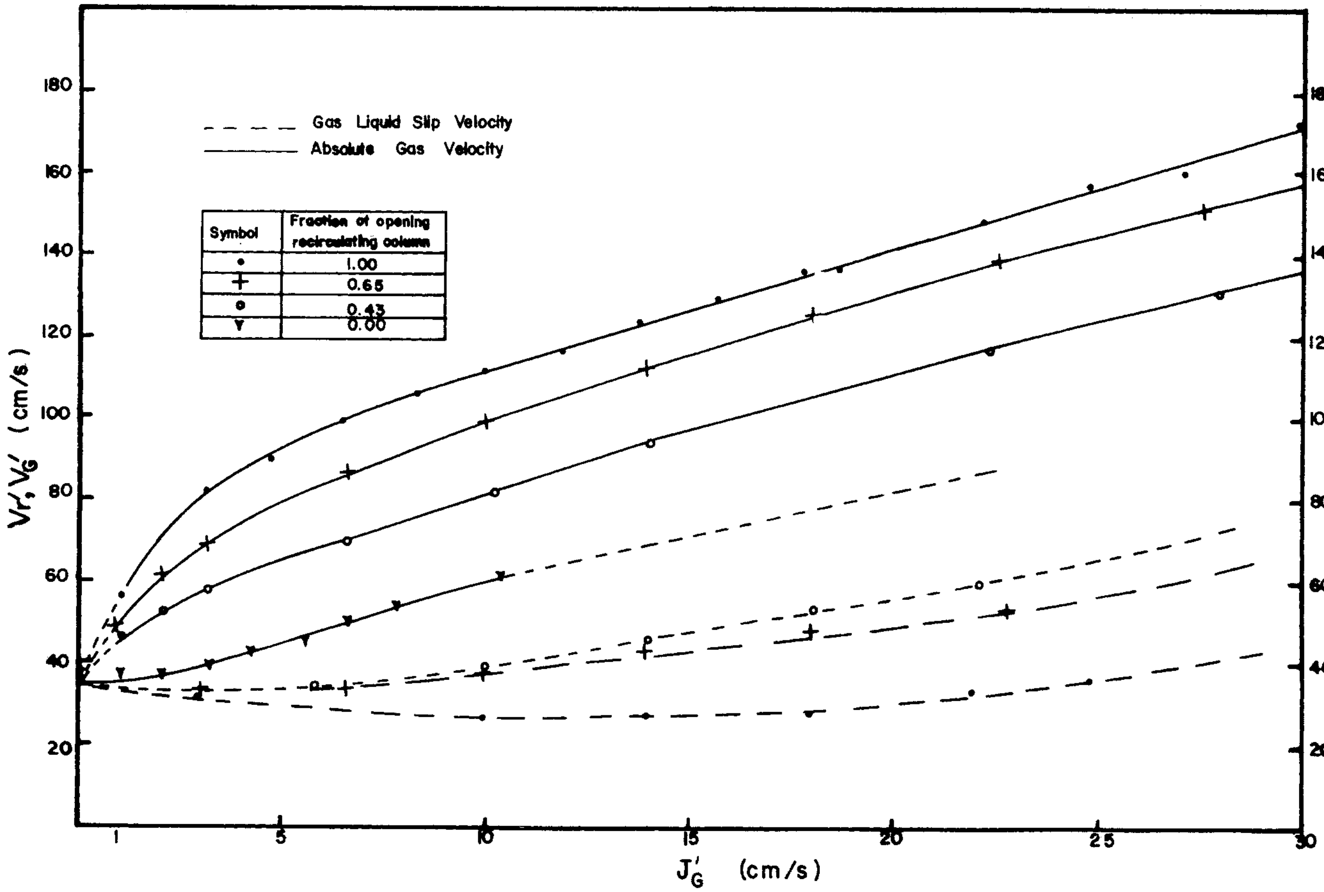

Figure 9. Gas absolute and relative velocity as a function of the gas superficial velocity at z = 2.25 m for various openings of the downcomer and sparger A.

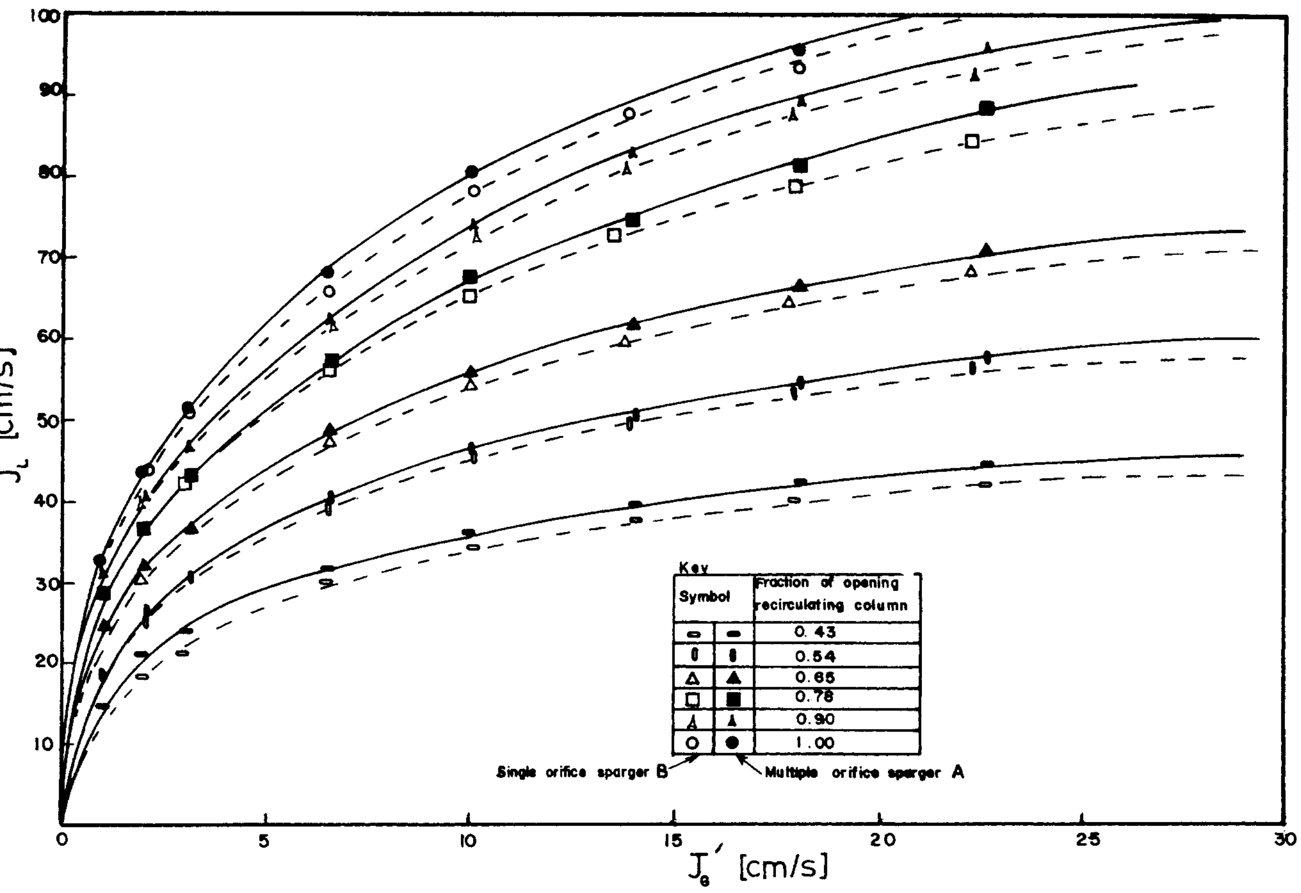

Figure 10. Superficial liquid velocity as a function of the superficial gas velocity for several openings of the downcomer and spargers A and B (J_G is taken at z = 2.25 m).

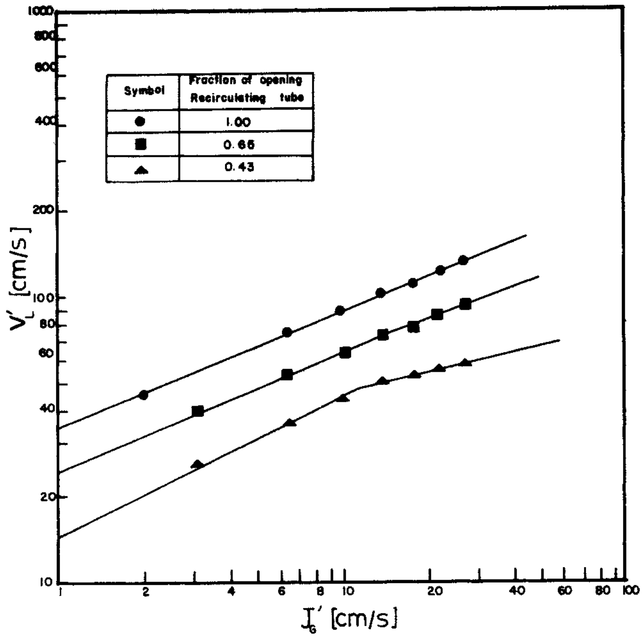

Figure 11. Liquid velocity as a function of the superficial gas velocity for various openings of the downcomer at z = 2.25 m. The values of n and b (Eq. 24) are as follows. Upper curve: n = 0.41, b = 34; middle curve: n = 0.41, b = 24; lower curve (low J'_G): n = 0.45, b = 13.6; lower curve (high J'_G or turbulent bubble flow): n = 0.22, b = 28.

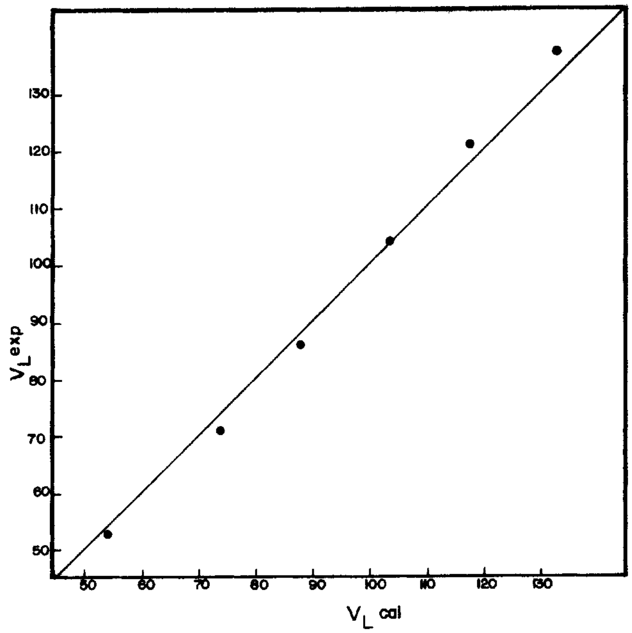

Figure 12. Comparison of measured values of V_L with those calculated with Eq. 33.

In the case of the experimental system used in this work, the assessment of the equivalent length is difficult because of the presence of the straightening vanes, sometimes closed in order to diminish the liquid circulation velocity. Therefore, L_E was determined from measurements of the pressure drop in the different sections of the circuit. Measurements of the pressure drop are easy to perform even in large configurations where direct measurements of the liquid flow rate are difficult and expensive. The equivalent length was independent of the gas flow rate and was found to be L_E = 92 m. The results of V_L obtained from Eq. 33 with this L_E compared satisfactorily with the measured values of V_L, as can be seen in Figure 12.

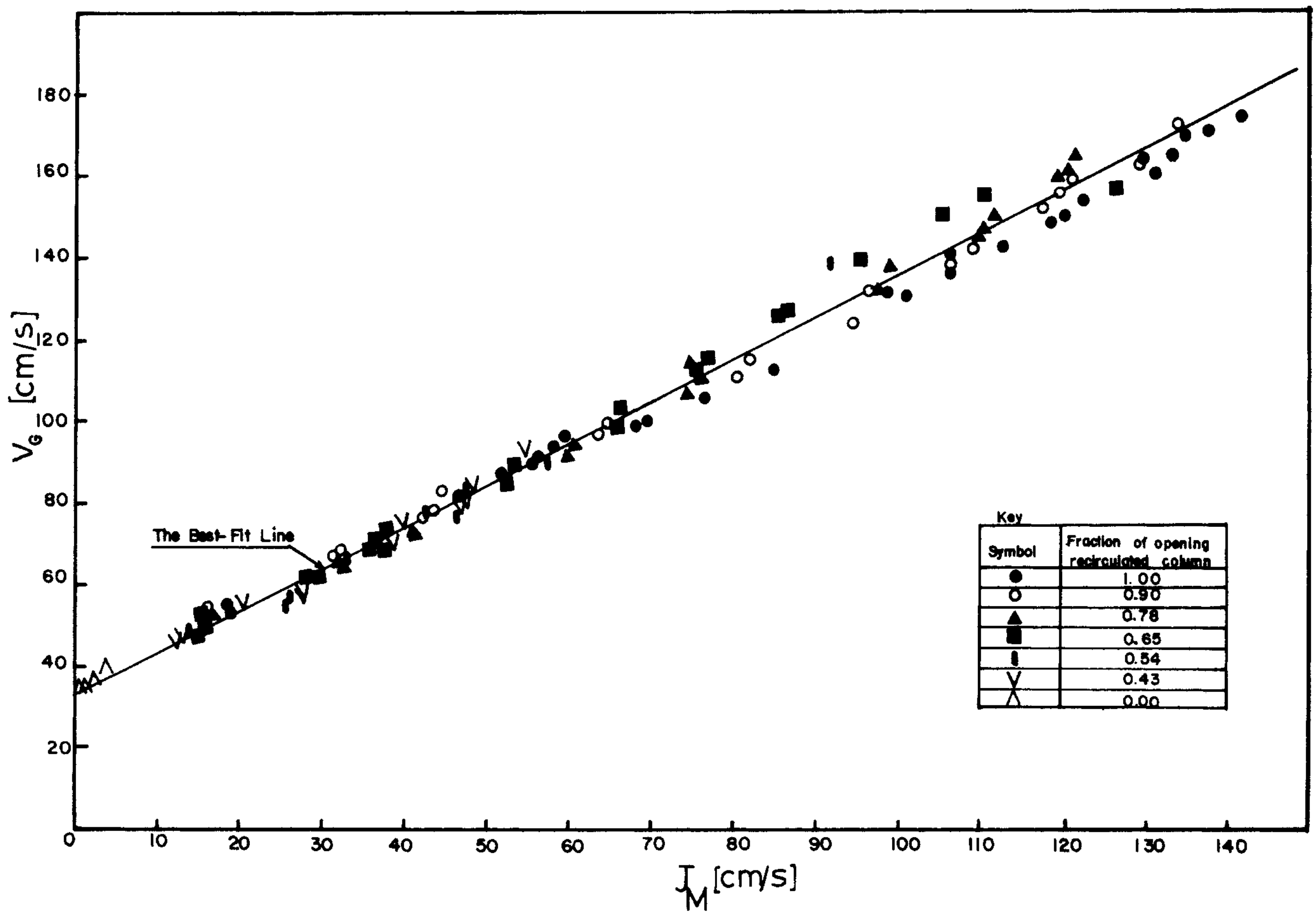

Figure 13. Gas velocity V_G vs. the overall flux of the mixture J_M for sparger A.

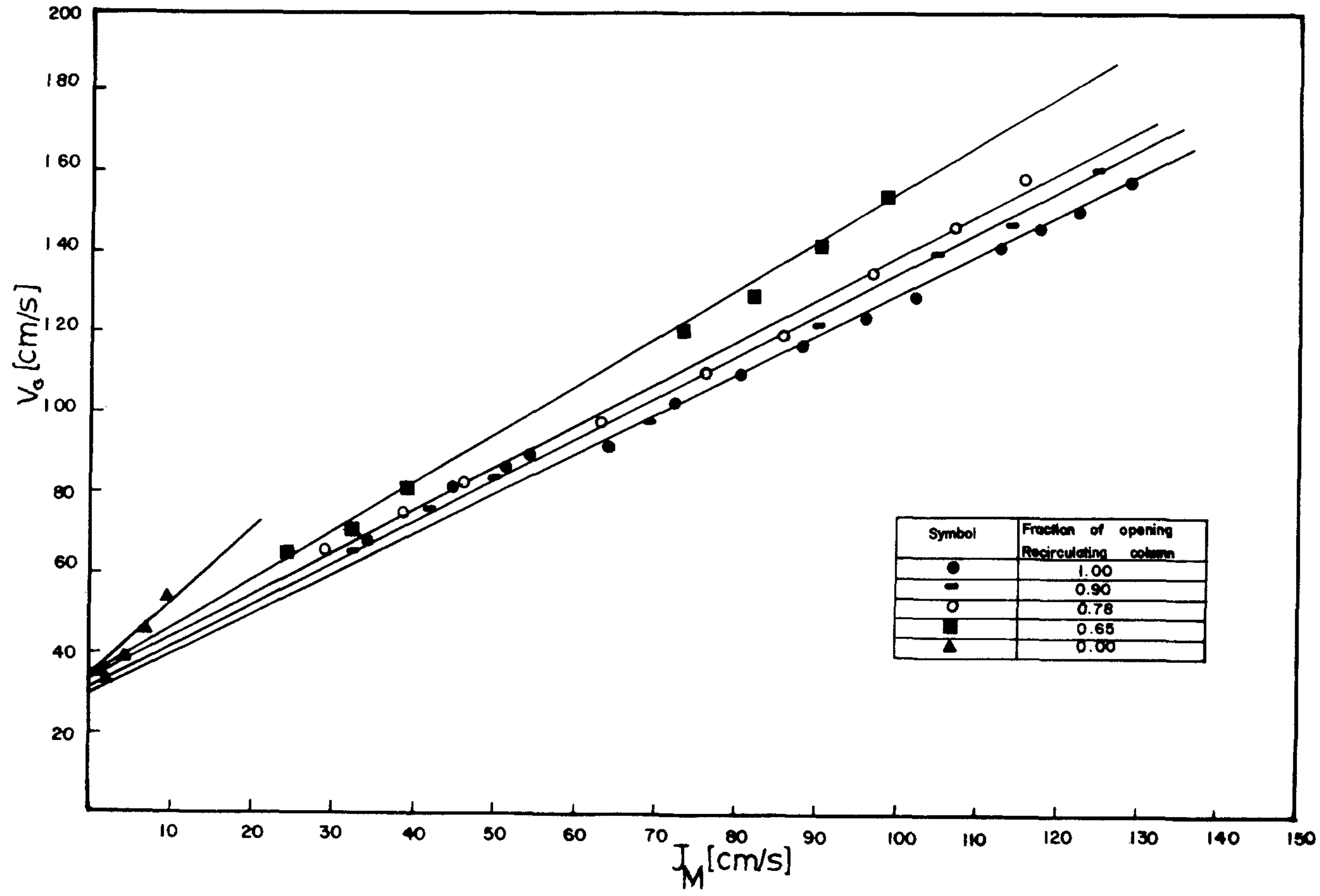

Figure 14. Gas velocity V_G vs. the overall flux of the mixture J_M for sparger B.

CORRELATION OF EXPERIMENTAL RESULTS

Zuber and Findley (1965) derived the following expression for the case of two-phase, one-directional flow:

$$V_G = \frac{J_G}{\phi} = C_oJ_M + V_D \tag{34}$$

The drift velocity V_D is the gas velocity relative to the velocity of the mixture, and C_o, the distribution parameter, is a constant that depends on the radial profiles of velocity and hold-up in the column. The flatter these profiles, the closer C_o will be to unity.

In Figure 13, the gas velocity V_G for sparger A is presented as a function of the overall flux of the mixture J_M for various locations along the height of the tower and various openings of the downcomer. All data are fitted satisfactorily by a single straight line, giving $V_D = 0.33$ m/s, which is in agreement with the value found for V_r at low superficial gas velocity.

The value obtained from the slope of the straight line in Figure 13 is $C_o = 1.03$, indicating fairly flat velocity and hold-up radial profiles all along the tower and for all experimental conditions with sparger A. The correlation reported in the literature for an air-lift device—the one given by Hatch (1973) for two concentric tubes—coincides formally with that reported here for separate tubes. Nassos and Bankoff (1967) and Hill (1976) reported similar results for separate tubes.

Figure 14 shows the data of V_G vs. the overall flux of the mixture J_M for the case of the single-orifice sparger B. In this case, the data corresponding to each fixed downcomer closure can be approximated by a different straight line, and no single curve can be said to represent all of the data. This difference between the results for spargers A and B seems to be related to the high coalescence rate and internal recirculation in sparger B.

SUMMARY

The experiments reported showed a marked difference in the hydrodynamics of the air-lift reactor when operated with a multiple-orifice or a single-orifice sparger. This indicates that even in a relatively tall system like the one tested, the initial bubble size and distribution on the section of the tube is very important.

The local hold-up varies up to 80% in some cases and, with the exception of the case of the multiple-orifice sparger and high liquid velocities, shows a maximum before reaching the top of the column. The location of this maximum comes closer to the gas sparger as the resistance to the liquid flow is increased and as J_L decreases. This phenomenon is more remarkable in the case of the single-orifice sparger and is associated with bubble coalescence, which provokes a higher mixture velocity in the axis of the tube where the larger bubbles tend to concentrate.

The mean gas hold-up depends on the sparger used, as well as on the resistance to liquid recirculation in the downcomer, which establishes the range of liquid velocities. The data suggest that pressure drop in the downcomer should be minimized if the plug flow is sought in the riser.

As suggested by Zuber and Findley (1965) and Bankoff (1960), a single straight line can represent satisfactorily the gas velocity V_G at any point of the column with either sparger used and at the entire range of operational conditions (Figure 13).

The liquid velocity can be represented by a simple exponential form, which includes two constants that are functions of the geometry of the system. If direct measurement of the liquid flow rate is not possible, it can be evaluated from relatively simple balances in the system, provided the pressure drops can be estimated or measured.

The liquid and gas velocities as well as the hold-up are important data, necessary for simulation, scale-up and design of an air-lift reactor. The present work contributes to the relatively scarce data available in the literature on this kind of device.

ACKNOWLEDGMENTS

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APPENDIX: ERROR ESTIMATION

The errors in the direct measurements are estimated as follows:

- Error in temperature reading, ±0.5°C
- Error in pressure reading at the entrance of the rotameter, ±0.01 kg/cm^2
- Error in pressure reading at the level of the sparger ($z = 0$) ±0.001 kg/cm^2
- Error in liquid flow rate reading, Q_L, ±3 L/min
- Error in gas flow rate, Q_G, ±1 L/min
- Error in height reading, z, ±0.5 cm
- Error in manometer reading, h, ±0.1 cm
- Error in diameter determination, D_c, ±0.05 cm

Assuming that the errors in the physical properties due to the 0.5°C uncertainty are negligible, Eq. 18 can be written as:

$$\phi = C_1' J_L^{0.8} J_M D_c^{-1.2} (1 - \phi)^{0.2} + C_2' J_L^2 (1 - \phi)^{-2} \frac{d\phi}{dz} + C_3' \frac{dh}{dz} \tag{A1}$$

where C_1', C_2' and C_3' are constants.

The term J_L can be obtained from

$$J_L = \frac{Q_L}{0.785 D_c^2} \tag{A2}$$

The calculation of J_G requires some manipulation of the measured quantities. Since the rotameter was calibrated at $P_R = 1$ atm and $T_R = 70$°F, and the pressure and temperature at the rotameter are P and T:

$$Q_G = Q_R \sqrt{\frac{TP_R}{PT_R}} \tag{A3}$$

Assuming ideal gas behavior at the sparger, the gas flow rate will be:

$$Q_{GS} = Q_G \frac{PT(z = 0)}{TP(z = 0)} \tag{A4}$$

Assuming the dry air is saturated almost completely near the sparger, the gas flow rate is increased by the flow rate of water vapor Q_V:

$$Q_G = [Q_{GS} + Q_V] \frac{P_w}{p} \tag{A5}$$

where p_w is the vapor pressure of water at this temperature. The gas flow rate at a point z is given by:

$$Q_G(z) = Q_{GS} \frac{P(z = 0)}{P(z)} \left[1 + \frac{p_w}{P(z = 0) - p_w} \right] \tag{A6}$$

and $P(z)$ can be calculated with:

$$P(z) = P(z = 0) - [\rho_L z - h(\rho_m - \rho_L)] g \tag{A7}$$

Eqs. A3 to A7 lead to the following expression:

$$Q_G(z) = \sqrt{\frac{PPr}{TTr}} \frac{T(z = 0) Q_R \left[1 + \frac{p_w}{P(z = 0) - p_w} \right]}{P(z = 0) - [\rho_L z - h(\rho_m - \rho_L)] 10^{-3}} \tag{A8}$$

Replacing J_L and J_G by Eqs. A2 and A8 leads to a function F that gives implicitly ϕ as a function of Q_L, Q_R, D_c, P, T, $T_{(z=0)}$, $P_{(z=0)}$, z and h:

$$F(\phi, Q_L, Q_R, D_c, P, T, T_{(z=0)}, P_{(z=0)}, z, h) = 0 \tag{A9}$$

The maximum error for a single determination was found as:

$$\Delta\phi = \Sigma \frac{\partial \phi}{\partial X_i} \Delta X_i \tag{A10}$$

where the X_i's are Q_L, Q_R, D_c, P, T, $T_{(z=0)}$, $P_{(z=0)}$, z, and h. The derivatives are obtained from the implicit function (Eq. A9) and the ΔX_i's are the errors in the direct measurements given above.

The value of h used in Eq. A8 is the direct measured height of the manometer at the point z. The derivatives of h and ϕ in Eq. A1 (the latter evaluated using Eq. 19) are functions of z, since the polynomial fitting of the data was used and the errors that originated in the polynomial approximation are included in the z term of Eq. A10.

The standard errors of estimation in the coefficients of the polynomial:

$$h = b_o + b_1 z + b_2 z^2 + b_3 z^3 + b_4 z^4 \tag{A11}$$

were calculated as follows. Let X be the matrix where the first column consists of ones, the second of values of z, the third column the values of z^2, etc. The covariance of b_i is found as:

$$cov \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \sigma^2 (X'X)^{-1} \tag{A12}$$

where σ is the standard deviation in the regression leading to Eq. A11.

An example of a typical calculation follows. The measured data are:

z	= 225 cm
Q_L	= 850 L/min
h	= 37.07 cm
T	= 19°C (gas temperature in the rotameter)
$P(z)$	= 0.74 kg/cm^2 (gauge) (local pressure)
$P(z = 0)$	= 0.434 kg/cm^2 (gauge) (bottom pressure)
ρ_m	= 1.457 g/cm^3 (density of manometric fluid)
Q_R	= 130.2 L/min (gas flow rate at the rotameter)
ρ_L	= 1.0 g/cm^3
$T(z = 0)$	= 18°C

The regression on the data of h gives:

σ = 2.12 cm
b_1 = 0.117 ± 0.010 cm^{-1}
b_2 = (0.227 ± 0.056)10^{-3} cm^{-2}
b_4 = (1.11 ± 0.22)10^{-9} cm^{-4}
$b_o = b_3 = 0$

The error calculated from Eq. A10 is:

$$\Delta\phi = 4.688 \; 10^{-3}$$

Since the value of hold-up obtained in this case is $\phi = 0.101$, the relative error is below 6.5%.

NOTATION

A	= cross-section of the tube, m^2
b	= constant in Eq. 24, (m/s)$^{1-n}$
C_f	= friction coefficient, (−)
C_{WG}	= contact area between the gas phase and the wall, m^2
C_{LW}	= contact area between the liquid phase and the wall, m^2
C_{LG}, C_{GL}	= contact area between the liquid phase and the gas phase, m^2
C_o	= distribution parameter, (−)
D_c	= tube diameter, m
D_h	= hydraulic diameter, m
g	= gravitational acceleration, m/s^2
h	= manometric differential reading, m
J	= superficial velocity, m/s
J'	= superficial velocity at the point z = 2.25 m, m/s
L	= tube length, m
L_e	= equivalent tube length, m
$\dot{m}$	= mass flow rate, kg/s
N_{Re}	= mixture Reynolds number, (−)
n	= constant in Eq. 24, (−)
P	= pressure, *Pa*
Q	= volumetric flow rate, m^3/s
V	= velocity, m/s
z	= axial coordinate, m

Greek Letters

μ	= viscosity, poise

ν = kinematic viscosity, stokes
ζ = ratio of cross-sectional area of riser and downcomer, (−)
ρ = density, kg/m^3
σ = superficial tension, N/m
τ = shear stress, *Pa*
ϕ = hold-up, (−)
ϕ' = hold-up at z = 2.25 m, (−)

Indices

B = bottom
d = downcomer
F = friction
G = gas
H = hydrostatic
L = liquid
m = manometer
M = mixture
W = wall of the pipe
r = riser, relative

LITERATURE CITED

Bankoff, S. G., "A Variable Density Single-Fluid Model for Two-Phase Flow with Particular Reference to Steam-Water Flow," *J. Heat Transfer*, Trans. ASME, Series C., **82**, 265 (1960).

Belfield, A. R., Jr., "Experimental Studies of Oxygen Transfer in a Split Cylinder Air-Lift," M.S. Thesis, University of Maryland (1976).

Calderbank, P. H., M. B. Moo Young, and R. Bibby, *Third European Symposium on Chemical Reaction Engineering*, Pergamon Press (1964).

Freedman, W. and J. F. Davidson, "Hold-up and Liquid Circulation in Bubble Columns," *Trans. Inst. Chem. Eng.*, **47**, 251 (1969).

Gasner, L. L., "Development and Application of the Thin Channel Rectangular Air-Lift, Mass Transfer Reactor to Fermentation and Waste-Water Treatment Systems," *Biotechnol. Bioeng.*, **XVI**, 1179 (1974).

Gow, P. G., J. D. Littlehailes, S. R. L. Smith and R. B. Walter, "Single Cell Protein Production from Methanol: Bacteria," *Single Cell Protein II*, S. R. Tannenbaum and D. I. C. Wang, eds., Massachusetts Institute of Technology Press, Cambridge, Mass. (1975).

Hatch, R. T., "Experimental and Theoretical Studies of Oxygen Transfer in an Air-Lift Fermentor," Ph.D. Thesis, Massachusetts Institute of Technology (1973).

Hatch, R. T., and D. I. C. Wang, "Oxygen Transfer in the Air-Lift Fermentor," First Chemical Congress of the North American Continent, Mexico City (1975).

Hatch, R. T., A. G. Belfield and G. Goldhahn, "Oxygen Transfer in the Split-Cylinder Air-Lift," Annual Meeting of the American Chemical Society, Chicago, Ill. (1977).

Hells, J. H., "The Operation of a Bubble Column at High Throughputs, I. Gas Hold-up Measurements," *Chem. Eng. J.*, **12**, 89 (1976).

Ho, Ch. S., L. E. Erickson and L. T. Fan, "Modeling and Simulation of Oxygen Transfer in Air-Lift Fermentors," *Biotechnol. Bioeng.*, **XIX**, 1503 (1977).

Kanazawa, M., "The Production of Yeast from n-Parafins," *Single Cell Proteins II*, S. R. Tannenbaum and D. I. C. Wang, eds., Massachusetts Institute of Technology Press, Cambridge, Mass. (1975).

Kuraishi, M., N. Matsuda, A. Terao, K. Kamibayashi, K. Tonomura and T. Fujii, "A Study on the Internal Structure Design of Air-Lift Fermentors in the Production of Methanol Single Cell Protein," *Microbial Growth on C_1 Compounds*, Society of Fermentation Technology, Osaka, Japan, p. 231 (1975).

Le Francois, L., C. C. Mariller and J. V. Mejane, "Effectionnements aux Procedes de Cultures Forgiques et de Fermentations Industrielles," Brevet d'Invention, France, No. 1102200 (1955).

Legrys, G. A., "Power Demand and Mass Transfer Capability of Mechanically Agitated Gas-Liquid Contactors and their Relationship to Air-Lift Fermentors," *Chem. Eng. Sci.*, **33**, 83 (1977).

Marucci, G., "A Theory of Coalescence," *Chem. Eng. Sci.*, **23**, 975 (1969).

Merchuk, J. C., Y. Stein and R. I. Mateles, "A Distributed Parameter Model of an Air-Lift Fermentor," *Biotechnol. Bioeng.*, **22**, 123 (1980).

Nassos, G. P., and S. G. Bankoff, "Slip Velocity in an Air-Water System under Steady State and Transient Conditions," *Chem. Eng. Sci.*, **12**, 661 (1967).

Orazem, M. E., and L. E. Erickson, "Oxygen-Transfer Rates and Efficiencies in One and Two-Stage Air-Lift Towers," *Biotechnol. Bioeng.*, **XXI**, 69 (1979).

Schugerl, K., J. Lucko and U. Oels, "Bubble Column Bioreactors," *Adv. Biochem. Eng.*, **7**, 1 (1977).

Stein, Y., "Gas Hold-up, Liquid Circulation, and Mass Transfer Modeling and Simulation of a Tower Cycling Fermentor," MS Thesis, Ben Gurion University, Israel (1979).

Ueyama, K., and T. Miyauchi, "Behaviour of Bubbles and Liquid in a Bubble Column," *Kagaku Kogaku Rombunshu*, **3**, 19 (1977a).

Ueyama, K., and T. Miyauchi, "The Effects of Liquid Viscosity and Column Diameter on the Internal Circulating Flow in a Bubble Column," *Kagaku Kogaku Rombunshu*, **3**, 115 (1977b).

Wallis, G. B., "One-Dimensional Two-Phase Flow," McGraw Hill Inc., New York (1969).

Wang, D. I. C., "Proteins from Petroleum," *Chem. Eng.*, **75**, 99 (1968).

Zuber, R. N., and J. A. Findley, "Average Volumetric Concentrations in Two-Phase Flow Systems," *J. Heat Transfer*, **87**, 453 (1965).

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Pressure Fluctuations in a Fluidized Bed

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An on-line statistical study of the pressure fluctuations in fluidized beds was conducted by using pressure transducers, a correlation and probability analyzer and a Fourier transform analyzer. The causes of the fluctuations were explored, and the effects of the gas velocity, bed height, particle size and distributor design on the major frequency and amplitude of the fluctuations were investigated. The results indicate that the motion of bubbles appears to be the major cause of the pressure fluctuations in the upper portion of a fluidized bed. In the lower portion, the combined effects of the formation of large bubbles in the middle portion of the bed, the formation of small bubbles near the distributor, and the jet flow immediately above the distributor appear to be the major causes of pressure fluctuations.

SCOPE

Pressure fluctuations have been observed to occur in most fluidized beds and these fluctuations have been used to define an index for the quality of fluidization (Shuster and Kisliak, 1952; Fiocco, 1964; Sutherland, 1964; Winter, 1968). Small and rapid fluctuations are considered to be associated with a good quality of fluidization.

The nature of pressure fluctuations in a fluidized bed is a complex function of particle properties, bed geometry, pressure in the bed, and properties and flow conditions of the fluidizing fluid. The pressure fluctuations have been studied by

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